State space representation:

$\lambda = L(x)i$

$V = iR + \frac{d\lambda}{dt} \Rightarrow \boxed{V = iR + L(x)\frac{di}{dt} + \frac{\partial L}{\partial x}\frac{dx}{dt}i}$

$f^e = \frac{1}{2}\frac{\partial L}{\partial x}i^2$

$\boxed{m\ddot{x} = f_s + f_D + f^e}$

Rewrite as time derivatives of the state variables:

State variables: tell how system is right now and can be used to predict where system will be in the future.

Mechanical: position (x) and velocity ($\dot{x}$)

Electrical: potential (v) and current (i)

KVL and Newton's 2nd Law:

$V = iR + L(x)\frac{di}{dt} + \frac{\partial L}{\partial x}\frac{dx}{dt}i$

$m\ddot{x} = f_s + f_D + f^e$

Rewrite in state-space form: state variables: $x, \dot{x}, i$ How do they change in time?

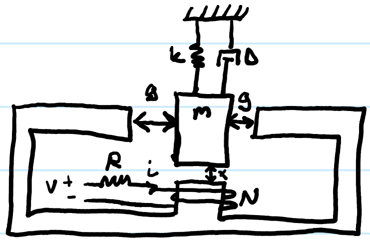
$V = iR + L(x)\frac{di}{dt} + \frac{\partial L}{\partial x}\frac{dx}{dt}i \Rightarrow L(x)\frac{di}{dt} = V - i\left(R + \frac{\partial L}{\partial x}\frac{dx}{dt}\right) \Rightarrow \boxed{\frac{di}{dt} = \frac{1}{L(x)}\left[V - i\left(R + \frac{\partial L}{\partial x}\frac{dx}{dt}\right)\right]}$

$\ddot{x} = \frac{d\dot{x}}{dt} \Rightarrow m\frac{d\dot{x}}{dt} = f_s + f_D + f^e \Rightarrow \boxed{\frac{d\dot{x}}{dt} = \frac{1}{m}\left(f_s + f_D + f^e\right)}$

$\boxed{\frac{dx}{dt} = \dot{x}}$

2018-11-05-2

Ex



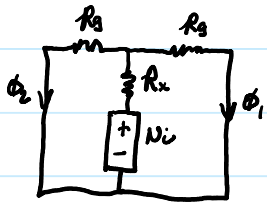
g is fixed

$A_x = A_g = A$

$f_s(x=\ell)=0$

$\mu_{iron} = \infty$

$V = V_0 = \text{constant}$



$R_x = \frac{x}{\mu_0 A} \qquad R_g = \frac{g}{\mu_0 A}$

$Ni = R_g\phi_1 + R_x(\phi_1+\phi_2)$

$Ni = R_g\phi_2 + R_x(\phi_1+\phi_2)$

$\Rightarrow \phi_1 = \phi_2 = \phi$

$Ni = (R_g + 2R_x)\phi$

$\phi = \left(\frac{\mu_0 A N}{g+2x}\right) i$

$\lambda = \left(\frac{\mu_0 A N^2}{g+2x}\right) i$

$$W_m' = \int_0^i \lambda\, di \Rightarrow W_m' = \frac{1}{2}\left(\frac{\mu_0 A N^2}{g+2x}\right) i^2$$

$$f^e = \frac{\partial W_m'}{\partial x} \Rightarrow f^e = \frac{1}{2}\left(\frac{-\mu_0 A N^2}{(g+2x)^2}\right)(2) i^2 \Rightarrow f^e = -\left(\frac{\mu_0 A N^2}{(g+2x)^2}\right) i^2$$

$$V = iR + \left(\frac{\mu_0 A N^2}{g+2x}\right)\frac{di}{dt} - \left(\frac{2\mu_0 A N^2}{(g+2x)^2}\right)\frac{dx}{dt} i$$

$$\Rightarrow \boxed{\frac{di}{dt} = \frac{g+2x}{\mu_0 A N^2}\left[V - i\left(R - \frac{2\mu_0 A N^2}{(g+2x)^2}\frac{dx}{dt}\right)\right]}$$

$$m\ddot{x} = -k(x-\ell) - D\dot{x} + f^e$$

$$m\ddot{x} = -k(x-\ell) - D\dot{x} - \left(\frac{\mu_0 A N^2}{(g+2x)^2}\right) i^2$$

$$\boxed{\begin{aligned} \frac{d\dot{x}}{dt} &= \frac{1}{m}\left[-k(x-\ell) - D\dot{x} - \left(\frac{\mu_0 A N^2}{(g+2x)^2}\right) i^2\right] \\ \frac{dx}{dt} &= \dot{x} \end{aligned}}$$

As seen in the previous example, the governing equations are a set of coupled non-linear ODE's. How to solve?

* Numerical approximations on the computer.

<u>Euler's Method:</u> Recall definition of the derivative: $\frac{dy}{dt} = \lim_{\Delta t \to 0} \frac{y(t+\Delta t) - y(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{y(t) - y(t-\Delta t)}{\Delta t}$

What happens if Δt doesn't go to zero?

<u>Taylor Series expand:</u> $y(t-\Delta t) = y(t) - \Delta t \frac{dy}{dt} + \Delta t^2 \frac{d^2y}{dt^2} - \ldots$

$$y(t) - y(t-\Delta t) = \Delta t \frac{dy}{dt} - \Delta t^2 \frac{d^2y}{dt^2} + \ldots$$

$$\frac{y(t) - y(t-\Delta t)}{\Delta t} = \frac{dy}{dt} - \Delta t \frac{d^2y}{dt^2} + \ldots$$

So, if $\Delta t \ll 1$, $\frac{dy}{dt} \approx \frac{y(t) - y(t-\Delta t)}{\Delta t}$

<u>Nomenclature:</u> Δt: time step (known)

$y(t)$: function at current time step (want to find)

$y(t-\Delta t)$: function at previous time step (known)

<u>Ex)</u> $\frac{dy}{dt} = f(t,y)$

$$\frac{dy}{dt} \approx \frac{y(t) - y(t-\Delta t)}{\Delta t}$$

* Can also show that: $f(t,y) \approx f(t-\Delta t, y(t-\Delta t))$

$$\frac{y(t) - y(t-\Delta t)}{\Delta t} = f(t-\Delta t, y(t-\Delta t))$$

$$\Rightarrow \boxed{y(t) = y(t-\Delta t) + \Delta t\, f(t-\Delta t, y(t-\Delta t))}$$

* This is called forward Euler method, and is explicit because the RHS is known.

2018-11-05-4

Ex| $\frac{dy}{dt} = -y$ $\quad\quad y(t=0)=1$

$$\frac{y(t)-y(t-\Delta t)}{\Delta t} = -y(t-\Delta t)$$

$$y(t) = y(t-\Delta t) - \Delta t\, y(t-\Delta t)$$

$$y(t) = (1-\Delta t)\, y(t-\Delta t)$$

set $\Delta t = 0.01\,s$

t	y
0	1
0.01	0.99
0.02	0.9801
0.03	0.9703

* Excel Sheet example.